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Engineering Mathematics Education in Australia

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Mathematics in Australia

Despite its relatively small population, Australia has a long tradition of producing strong mathematicians; in many of the world's great mathematics departments, you will hear an Australian accent. This tradition has been underpinned by a high quality system of universal education at the school level. However, over the last 12 years, there has been a decline in student and staff numbers in several university mathematics departments. At the same time, there has been a growing shortage of teachers qualified in the subject. The quality of mathematics education is not as uniform as it once was.

During 2006, there was a national review of mathematical sciences in Australia [1]. Prompted by the Australian Research Council, this was originally planned to be a review of mathematical sciences research. However it soon became evident that the issues around mathematics education were threatening research productivity.

> ***"Australia's distinguished tradition and capability in mathematics and statistics is on a truly perilous path."*** [2]

Over the past 10 years the mathematical sciences disciplines have been unintended victims of changes in discipline-based funding in higher education, relaxation of Year 12 mathematics prerequisites for professional degree courses, and a general cultural shift away from the physical sciences. As a result, mathematics and statistics departments at Australian universities are gradually being eroded. Between 1996 and 2006 around a third of academic positions in university mathematical sciences departments had been lost [1], with many universities now employing fewer than 10 mathematics staff.

The implications of this loss of mathematical expertise are far reaching; Australian industry can only compete internationally by clever design, efficiency, process control and quality control, all of which are inherently mathematical. Australia's skilled workforce is now ageing with insufficient numbers of young mathematicians and statisticians to take their place. In 2003 only 0.4% of Australian university students graduated with mathematical qualifications compared with the Organisation for Economic Co-operation and Development (OECD) average of 1% [3]. The number of mathematics major graduates continues to decline [4] yet demand for their skill is steadily increasing (Fig 1). The Department of Education Science and Training's (DEST) Science, Engineering and Technology Skills Audit [5] shows that the period 1997-2005 has already experienced a 52% growth in employment demand for the mathematical sciences, compared to 37% over all natural sciences. At the same time, a review of science education by the Australian Council of Deans of Science (ACDS) [6] shows that the number of university science students taking a mathematics subject has decreased by 34 %.

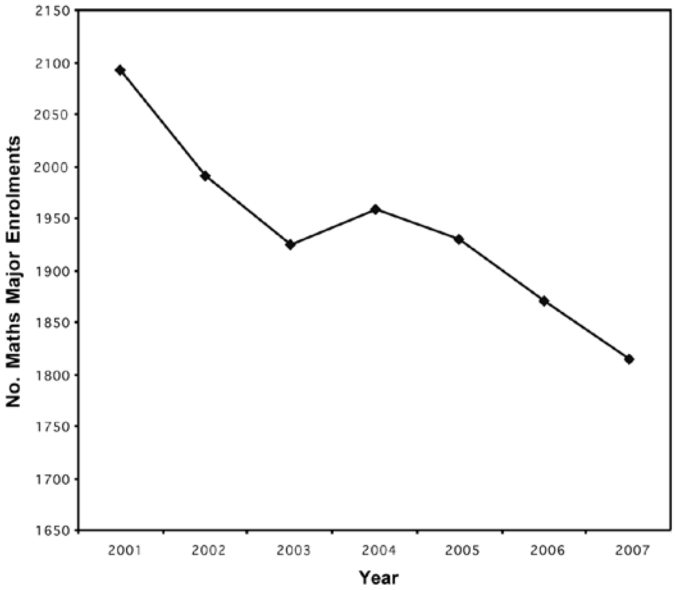

Fig 1: Number of Mathematics Major Student Enrolments 2001-2007 [4]

With this background, the Australian Mathematical Sciences Institute (AMSI), with funding from the former Carrick Institute of Learning and Teaching (now the Australian Learning and Teaching Council (ALTC)) undertook a project to examine practices in mathematics education for 21st century engineering students [7].

Mathematics in engineering degree programs

The current version of the Engineers Australia (EA) curriculum guideline [8], while fully compliant with the Washington Accord [9], gives no specific advice on mathematics content. Mathematics is included as part of a section of the indicative components of the total learning experience:

> ***"Mathematics, science, engineering principles, skills and tools appropriate to the discipline of study (not less than 40%)"***

and is again mentioned under enabling skills and knowledge development:

> ***"Enabling skills and knowledge in mathematics; physical, life and information science, and in engineering fundamentals must adequately underpin the development of high level technical capabilities, and in engineering application work…"***

There is some commonality in core mathematics content at first year level. This project [7], and an independent study [10] found widespread uniformity in the coverage of topics of calculus (introductory and one dimensional), linear algebra, multivariable calculus, ordinary differential equations and introductory statistics. In Australia, introductory calculus is commonly integrated with some algebra, probability, discrete mathematics, numerical analysis, and complex analysis. The amount of material in the last four of these topics varies between institutions.

There is also some uniformity in introductory probability [10]. However, several engineering students and staff have stressed the need for more probability and statistics in response to modern engineering practice [11, 12]. Larger capital city universities tend to run four compulsory mathematics subjects for engineering students while the smaller universities tend to run two or three; hence coverage is not as broad.

Although the report found wide recognition that different specialisations of engineering favour different mathematics topics, distinct first year mathematics subjects were offered to distinct specialisations at only three institutions. Some institutions offer the same first year mathematics not only to engineering majors, but to all mathematics and science majors. One significant motivation for this is increased pressure to produce more research outputs.

Challenges in learning and teaching

Most respondents [7] perceived a decline in the mathematical preparation of new students, with many attributing this to school teaching standards, lowering of entry standards and alternative entry routes. Most identified this decline as the greatest challenge faced.

Year 12 mathematics curricula vary among the eight states and territories. Students at that level commonly take five or six subjects. The great majority take one mathematics subject, with some taking an additional subject at an advanced level. Three levels of mathematics subjects are offered. These are classified [13] as being elementary, intermediate and advanced. Calculus is integrated with other topics but as a rule, it is absent in elementary courses, present in a bare introductory form in intermediate mathematics and extended to a more rigorous form, with some applications in the advanced courses. While overall participation in school mathematics appears to be healthy and even to have increased over the past 10 years,

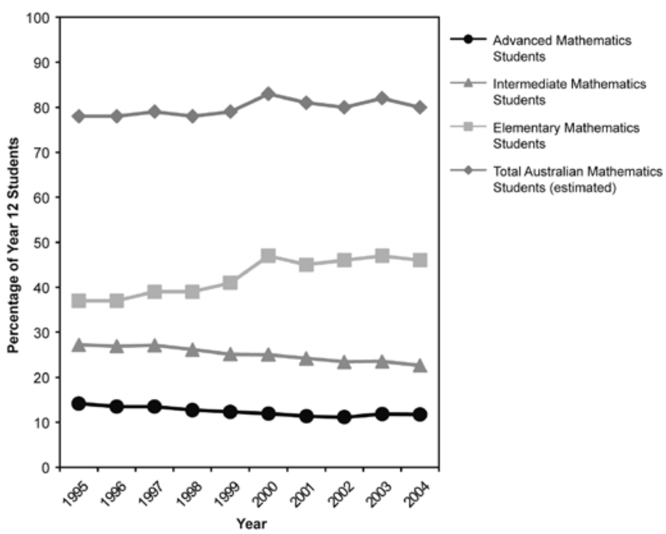

Fig 2: Year 12 Mathematics Students as Percentages of Year 12 Students [14]

enrolments in advanced and intermediate mathematics are steadily declining in favour of elementary mathematics [14] (Fig 2). Only 64% of high schools now offer advanced mathematics [15]. There is a widespread, perhaps erroneous belief that students can improve their Universities Admission Index (a national index that ranks students from their state assessments) by taking less demanding subjects.

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A Senate Committee Report [16] found the source of problems of declining standards to be "…at the bottom of the school and the top: the failure to instil the required level of 'numeracy' in the primary school years; and the failure to encourage the required degree of rigour in a larger proportion of students in the senior secondary years". Significant variation was found in the Year 12 mathematics subjects between the states [14]. Respondents [7] expressed concerns about changes to secondary curricula and in differences between states. The present government has set up a National Curriculum Board that will eventually prescribe some common school mathematics content.

With continual pressure on departments to increase enrolments, most have dropped the advanced mathematics requirement. Engineering students entering with advanced mathematics are seen as highly desirable; a disproportionate number of them enrol in the three oldest universities. The increase in international enrolments is further broadening diversity of students' mathematical backgrounds.

In 2004 more than 27% of new Bachelor of Engineering (BE) student enrolments were international students (a significant increase from 13% in 1996) while Australian commencing engineering enrolments have fluctuated around 10,000 for the same period [17]. Fuelled by a boom in the minerals sector, the engineering profession aims to increase domestic BE enrolments. Nationwide, only around 80,000 students take intermediate or advanced Year 12 mathematics [14].

Most respondents [7] when asked about challenges in learning and teaching, discussed the decrease in incoming students' ability, the reduction in mathematics subjects, content and teaching hours, the lowering of entry standards and the diversity of students' backgrounds. Highlighting the inadequacy of the "one size fits all" approach to mathematics for engineering students, some commented on the difficulty of attempting to cater for the mathematical needs of all engineering disciplines in one subject and reaching a shared understanding between the mathematics and engineering departments of what is to be included in the curriculum. Comment was also made about the difficulty of engaging students, with many demanding to know immediately the relevance of everything that is taught.

Addressing the challenges

There has been a significant extension of both academic and pastoral student support services in universities. Mathematics support is offered for engineering students in a variety of forms, often by way of peer assisted learning. Some institutions operate mathematics learning centres, manned by staff or postgraduate students to provide a variety of resources for students. Increased office hours and extra tutorials are also offered by staff, usually on a drop-in basis. This additional support has various degrees of access, ranging from restricted to low-achieving students, to voluntary attendance available to all. Most student feedback regarding mathematics support has been positive. Higher-level engineering students have been seen to be particularly motivating to first and second year students as they are able to reiterate, from recent experience, the value of good mathematical foundations in more senior courses.

Educators have explored ways to incorporate topics in professional practice without sacrificing technical content, often including group work in technical subjects. Supporting the need for the acquisition of these skills in conjunction with theoretical learning throughout the engineering degree program, Trevelyan [18] found that engineers in the workplace do "lots of co-ordination in which technical knowledge is inextricably bound up with 'soft skills' and understanding of human behaviour". Lopez [19] also found a great deal of research highlighting the benefits of group learning, including the ability to cater to different learning styles. Consequently problem- and project- based learning are now widely used in engineering programs and increasingly as a component of mathematics subjects, often in tutorials using engineering problems to illustrate the application of mathematical techniques. Interestingly there is some research that shows while students appreciate in-context examples, they do not like mathematics to be taught completely in-context, finding it difficult to transfer the principles to different contexts [20, 21, 22].

Advances in information technology and the improved affordability of computer hardware have meant that software applications and the world wide web have become integrated into daily life. Many authors reason that computer based methods in learning and teaching stimulate interest and enhance comprehension [19]. Online quizzes through learning management systems, commercial or in-house software are increasingly being used in the teaching of engineering mathematics. Many respondents identified the need for these resources because of decreased numbers of mathematics staff and increased class sizes making it difficult for lecturers and tutors to provide detailed and timely feedback to students. There are a variety of software programs available, with many institutions having developed their own. Questions vary from multiple choice to questions with multiple parts that must be entered as algebraic expressions. Similarly the level of feedback varies from binary correct/incorrect to in-house software providing targeted feedback on individual answers explaining why the answer is incorrect and suggesting further reading.

Consistent with the findings of Lopez [19], MATLAB was reported to be the most commonly used software at Australian universities. A number of innovative computer aided learning and assessment programs have been developed independently at various institutions both internationally and nationally (e.g. STACK [23] and CalMaeth [24]). A co-ordinated approach to the development of such software and a database of shared in-context questions would be invaluable. Many respondents commented on the difficulty of finding good in-context engineering examples and the resource constraints in providing feedback to students.

There is much debate as to the effectiveness of developmental mathematics programs. Most research concludes that these programs can be effective and beneficial to students. Most Australian universities now stream engineering mathematics students. The most common criterion used is level of high school mathematics taken. However, with the increase in international student numbers and variations between state curricula, this is becoming increasingly difficult. The streaming usually takes place only in first year, with all students expected to reach the same competency level to proceed to a common second year mathematics subject. Some institutions have now introduced their own diagnostic tests, often to advise students, but not compel students to a stream. Mathematics bridging courses are offered at a number of universities to be completed by students in the summer before commencing first year; often enrolment in these is not enforced. Alternative entry pathways are also being offered. EA [17] reported that students find the transition from colleges of Technical And Further Education (TAFE) to an engineering program difficult due to the lack of mathematics in certificate and diploma programs. Ex TAFE students commented that the mathematics is "daunting and difficult". A well developed standardised diagnostic test available to all universities would greatly assist in identifying students who require additional instruction.

Moving forward

In March 2007, a federal budget increase of AU$2,729 (approx. £1,200) was given to universities per equivalent full time university mathematics student (equating to approximately a AU$900,000 (approx. £400,000) increase in funding for a typical university). However, a national questionnaire [25] of the Australian Council of Heads of Mathematical Sciences (ACHMS) in February 2008 revealed that at least 50% and as much as 80% of this money allocated to the national priority disciplines appears to have been retained for administration. Despite the increased funding, there were almost 40 fewer mathematics teaching and research staff at the start of this year compared with 2007.

In March 2008, the federal budget halved Higher Education Contribution Scheme (HECS) payment (compulsory national fees) for domestic mathematics and science students. Also in March 2008, proposed staff cuts in the Department of Mathematics and Computing at the University of Southern Queensland (USQ) and the elimination of all non-service mathematics classes, sparked an international campaign [26].

The supply of adequately trained primary and secondary teachers is crucial in solving the problem of underperformance in school mathematics. However, only four out of 31 Australian universities require Year 12 mathematics as a prerequisite for Bachelor of Education courses and of the remainder only eight require Year 11 mathematics [27]. A report for the ACDS [15] found that one in four senior mathematics teachers do not undertake any third year mathematics study at university, one in 12 mathematics teachers studied no mathematics at university and one in five studied no mathematics beyond first year, concluding that these figures will only worsen with increasing retirements and lower university mathematics enrolments. Pleasingly, since the advent of the National Curriculum Board, dialogue between teachers' associations and mathematics professional bodies has improved to the extent that common goals are being formulated.

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Most academic and professional engineers agree that it is essential for engineers to have a good grounding in mathematics, including general logic and problem solving. There is growing concern that the majority of professional engineers in Australia are not confident to use mathematics in their careers. This is compounded by the widening gap between skills and knowledge acquired at secondary school and assumed knowledge on entry into first year engineering programs and the reduction of the number of mathematics subjects. Since the release of the report, a concerted effort has begun to collect good examples of real uses of mathematics in the engineering context. For example, the Australasian Association of Engineering Education (AaeE) and the Engineering Mathematics Group of the Australian Mathematical Society (AustMS) have been jointly planning a special issue of the Australasian Journal of Engineering Education. The report has been used to inform a second ALTC funded project for the Australian Council of Engineering Deans (ACED) to address issues of engineering education.

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Recommendations

Effective practice and innovations around the country must be shared and developed in order to achieve systemic improvement. It is crucial to achieve effective course design and delivery for the increasingly diverse body of students, with agreement in the selection of mathematics topics in the curriculum. In order to achieve this, mathematics prerequisites must not be reduced any further and those without the required knowledge must take an additional developmental subject. Mathematical assistance, in the form of help centres, should be available for all engineering mathematics students. Steps must be made to strengthen school mathematics and to encourage students to take higher level mathematics at Year 12.

For effective curriculum design, communication channels must be open between engineering and mathematics departments; a joint mathematics curriculum committee comprising representatives from both departments aids this and allows joint ownership in decisions about content.

It is important for practical problem solving capability, innovative design and underpinning engineering science research that at least a small number of engineering graduates are extensively trained in mathematical modelling and mathematical methods. A quantitative stream consisting of at least five mathematics subjects (significantly more than the current average of 3.5 and comparable to the minimum engineering mathematics requirements for BE degrees in US research universities) should be made available. A single one-semester optional subject in statistics and stochastic modelling should also be made available to all engineering students (in addition to existing mathematics subjects) due to the emerging requirements for these skills in engineering practice.

It is important that efficient methods are used to monitor progress and provide feedback to large classes; on-line formative assessment should be investigated to allow students to achieve the required level of rigour with rapid feedback. To improve the motivation of students, mathematics should be better related to the engineering context. It is therefore important that internal financial allocation systems are modified so that no budgetary unit is penalised for taking part in genuine multidisciplinary collaborative teaching. Mathematics departments in BE or ME-awarding institutions should identify staff with knowledge of engineering applications and allocate engineering mathematics teaching to them. A central bank of good examples and formative test questions, computer laboratory projects and curriculum resources should be developed by collaboration between engineering and mathematics teaching departments.

Mathematics and engineering departments must recognise that these challenges are shared as a national problem, requiring sharing of ideas, joint development of learning and assessment materials and joint strategies.

Acknowledgement

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References

1. National Committee for the Mathematical Sciences of the Australian Academy of Science (2006) "Critical Skills for Australia's Future". Accessed via: www.review.ms.unimelb.edu.au
2. Bourguignon, J. P., Dietrich, B. and Johnstone, I. M. (2006) "Foreword by the international reviewers" in National Committee for the Mathematical Sciences of the Australian Academy of Science (2006) "Critical Skills for Australia's Future". Accessed via: www.review.ms.unimelb.edu.au
3. Organisation for Economic Co-operation and Development (2004) "Education at a Glance – OECD Indicators". Accessed via: www.oecd.org
4. Thomas, J., Muchatuta, M and Wood, L. (2008) Mathematical Sciences in Australia. *International Journal of Mathematical Education in Science and Technology* (in press)
5. Department of Education, Science and Training (2006) "Audit of Science, Engineering and Technological Skills". Accessed via: www.dest.gov.au/sectors/science_innovation/policy_issues_reviews/key_issues/setsa/documents/summary_report_pdf.htm
6. Dobson, I. (2007) "Sustaining Science: University Science in the Twenty-First Century". A study commissioned by the Australian Council of Deans of Science. Accessed via: www.acds.edu.au/docs/DeansOfSci_FINAL.pdf

7. Broadbridge, P. and Henderson, S. (2008) "Mathematics Education for 21st Century Engineering Students: Final Report". Accessed via: www.amsi.org.au/21Engp.pdf

8. Engineers Australia (2006). "Accreditation Criteria Guidelines". Accessed via: www.engineersaustralia.org.au/shadomx/apps/fms/fmsdownload.cfm?file_uuid=0B19D0FF-0BC5-BAC1-DB36-6FB8599DDE67&siteName=ieaust

9. http://www.washingtonaccord.org/Washington-Accord/Accredited.cfm

10. Barry, S. and Healy, W. (2007) "Preliminary Study on Undergraduate Engineering Mathematics in Australia". School of Physical, Environmental and Mathematical Sciences, University of New South Wales, Australian Defense Force Academy

11. King, R. (2007) Mathematics for Engineers: observations from the Review of Engineering Education. *Proceedings of National Symposium on Mathematics Education for 21st Century Engineering Students. Melbourne, Australia*. Accessed via: www.amsi.org.au/carrick_seminar_program.php

12. MacGillivray, H. (2007) The multi-layered challenges of teaching statistics in engineering courses. *Proceedings of the National Symposium on Mathematics for 21st Century Engineering Students. Melbourne, Australia*. Accessed via: www.amsi.org.au/carrick_seminar_program.php

13. Barrington, F. and Brown, P. (2005) "Comparison of Year 12 mathematics subjects in Australia 2004-2005". Accessed via: www.amsi.org.au/pdfs/comp_y12pretertiary_aus_200404-05.pdf

14. Barrington, F. (2006) "Participation in Year 12 Mathematics across Australia 1995-2004". Accessed via: www.amsi.org.au/pdfs/Participation_in_Yr12_Maths.pdf

15. Harris, K. L. and Jensz, F. (2006) "The preparation of Mathematics Teachers in Australia". Report prepared for the Australian Council of Deans of Science. Accessed via: www.acds.edu.au/docs/Prep_Math_Teach_Aust.pdf

16. Senate Committee Report (2007) "Quality of School Education". Accessed via www.aph.gov.au/Senate/committee/eet_ctte/academic_standards/report/index.htm

17. King, R. (2008) "Addressing the Supply and Quality of Engineering Graduates for the New Century". Accessed via: www.altc.edu.au/carrick/webdav/users/siteadmin/public/Grants_DBIprojec_engineeringquality_project%20report_25march08.pdf

18. Trevelyan, J. (2008). Random Madness – Technical Co-ordination in Engineering Practice. *The Engineering Essential*. Accessed via: http://www.news.uwa.edu.au/jul-2008/

19. Lopez, A. (2007) (2008) "Mathematics Education for 21st Century Engineering Students: Literature Review". Accessed via: www.amsi.org.au/LitReviewW.pdf

20. Britton, S., New, P., Sharma, M., and Yardley, D. (2005) A case study of the transfer of mathematical skills by university students. *International Journal of Mathematical Education in Science and Technology*, Vol. 30 (No. 1):1-13.

21. Roberts, A., Sharma, M., Britton, S. and New, P. (2007) An index to measure the ability of first year science students to transfer mathematics. *International Journal of Mathematical Education in Science and Technology*, Vol. 38 (No. 4):429-448.

22. Wood, L. (2003) "Reflections Across Faculties (Volume 1 & 3)". Department of Mathematical Sciences, The University of Technology Sydney.

23. Sangwin, C. (2006). "Assessing Elementary Algebra with STACK". Accessed via: www.open.ac.uk/opencetl/resources/details/detail.php?itemId=461cdcef52913

24. Judd, K. (1996). "Teaching Intermediate Calculus by Computer". Accessed via: https://calmaeth.maths.uwa.edu.au/doc/reports/report1996.html

25. Thomas, J. (2008) Mathematical Sciences Questionnaire Report. Accessed via: www.amsi.org.au/pdfs/Questionnaire_summary.pdf

26. http://terrytao.wordpress.com/support-usq-maths/

27. Australian Mathematical Sciences Institute (2006) "Submission to House of Representatives Standing Committee on Education and Vocational Training *Inquiry into Teacher Education*". Accessed via: http://aph.gov.au/house/committee/evt/teachereduc/subs/sub058.pdf